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LETTER TO THE EDITOR

The absence of charge backscattering in the nonequilibrium current of normal–superconductor structures

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Abstract. We study the nonequilibrium transport properties of a normal–superconductor–normal structure, focusing on the effect of adding an impurity in the superconducting region. Current conservation requires the superfluid velocity to be nonzero, causing a distortion of the quasiparticle dispersion relation within the superconductor. For weakly reflecting interfaces we find a regime of intermediate voltages in which Andreev transmission is the only permitted mechanism for quasiparticles to enter the superconductor. Impurities in the superconductor can only cause Andreev reflection of these quasiparticles and thus cannot degrade the current. At higher voltages, a state of gapless superconductivity develops which is sensitive to the presence of impurities.

During the last few years, considerable progress has been made in the understanding of transport in mesoscopic structures with superconducting elements [1]. The possibility of phase-coherent Andreev reflection [2] provides a rich variety of situations with novel transport anomalies [3]. It has been pointed out [4, 5] that a limitation of conventional descriptions of Andreev reflection is that, by assuming a uniform phase in the superconducting region, they implicitly contain a violation of current conservation. While this shortcoming may be unimportant in the regime of low current densities, it certainly cannot be overlooked in situations where the superconductor supports current densities that are comparable to its equilibrium critical value. This may easily be the case in transmissive structures with applied voltages $V \sim \Delta_0/e$, where Δ_0 is the zero-current superconducting gap. It has been shown [4, 5] that the introduction of a nonuniform superconducting phase to ensure self-consistency (and, with it, charge conservation [6, 7, 8]) may have major consequences in the current–voltage characteristics of NS and NSN structures. Structures with weakly reflecting interfaces present a regime of intermediate voltages in which Andreev transmission (whereby an electron is transmitted as a quasihole, or vice versa) is the only allowed mechanism for quasiparticles to enter the S region. Here we wish to analyse this transport regime (hereafter referred to as the AT regime), which is characterized by the exclusive presence of Andreev transmitted quasiparticles. In particular, we report on the existence in the AT regime of a curious effect by which impurities are unable to degrade the quasiparticle electric current in the superconducting region. The origin of this effect is the strong distortion that the quasiparticle dispersion relation suffers in the presence of a condensate with a large superfluid velocity v_s, as is shown schematically in the inset of figure 1(*a*). The energy thresholds for quasiparticle propagation become $\Delta_\pm = \Delta_0 \pm \hbar v_F q$

for $k \simeq \pm k_F$, with $k_F = m v_F / \hbar$ the Fermi momentum and $q = m v_s / \hbar$. In an NSN structure, at zero temperature, and if both NS interfaces are identical, electrons (holes) come from the left (right) N lead with energies $0 < \varepsilon < eV/2$, and the AT regime is characterized by the condition $0 < \Delta_- < eV/2 < \Delta_+$, where V is the total voltage difference between the two N leads. At these intermediate voltages, the only scattering mechanisms available for electrons coming from the left N lead are normal reflection (NR), Andreev reflection (AR), and Andreev tranmission (AT). A similar consideration applies for holes coming from the right N lead. The current in S is then carried by the condensate and by quasiparticles lying exclusively in the low-energy branch of the asymmetric $\varepsilon(k)$ curve. One such quasiparticle (i.e., a right-moving quasihole) may undergo elastic scattering by impurities, with the peculiarity that, at energies $\Delta_- < \varepsilon < \Delta_+$, the only available scattering channels are either normal transmission (NT) as a right-moving quasihole or AR as a left-moving quasielectron [9]. It is clear that in both cases the electric current is left intact (up to corrections of order Δ_0/E_F). We conclude that, in the AT regime of intermediate voltages, the quasiparticle component of the electric current is essentially insensitive to the presence of impurities. In this article we present numerical results for a one-dimensional model that support this qualitative prediction. We also argue that the main conclusion concerning the absence of quasiparticle current degradation must apply under more realistic circumstances.

We solve the Bogoliubov–de Gennes equations [10] for a chosen one-dimensional structure with the requirement that the self-consistency condition

$$\Delta = g \sum_\alpha u_\alpha v_\alpha^* (1 - 2 f_\alpha) \tag{1}$$

is satisfied ((u_α, v_α) and f_α are the spinor wave function and the occupation probability for quasiparticle α). A number of assumptions are similar to those of [4]. The gap function is taken to be of the form $\Delta(x) = |\Delta| e^{2iqx}$, where the parameters $|\Delta|$ and q are determined self-consistently from equation (1) and from the conservation of current. Details of the numerical calculation will be given in a forthcoming publication [11]. The gap amplitude $|\Delta|$ is assumed to be uniform within the superconductor. This is expected to be a reasonable approximation for structures longer than a few times the zero-temperature coherence length $\xi_0 = \hbar v_F / \Delta_0$, so that $|\Delta(x)|$ can relax to its bulk saturation value [5]. We introduce one-electron delta function barriers of strength Z at the NS interfaces [4, 5, 12] and one barrier of strength Z_I to model the impurity within the superconductor [13]. We assume incoherent (albeit elastic) multiple scattering by the impurity and at the interfaces. This requires the existence of some type of phase breaking mechanism involving negligible energy loss for the quasiparticles. This set of simplifying assumptions allows us to focus on the physics introduced by the combined requirements of current conservation and impurity scattering at high current densities. The main physical effects come from the splitting of thresholds (from Δ_0 to $\Delta_\pm$ as v_s becomes nonzero), which gives rise to a variety of transport regimes.

We concentrate on the case of two moderately transmissive interfaces ($Z = 0.5$), since it is in this case where the different transport regimes can be most clearly identified [14]. The case of $Z = 0.5$ and $Z_I = 0$ has already been studied in [4] and we only review the main facts here. The AT channel opens for $v \equiv eV/\Delta_0 \simeq 1.3$, as can be seen from the jump in the total current (see figure 1(a) and from the onset of a negative quasiparticle current (figure 1(b)). One has $I_{qp} < 0$ because, in the available energy range ($\Delta_- < \varepsilon < eV/2 < \Delta_+$), the only quasiparticle channels are either right-moving holes or left-moving electrons, all contributing negatively to the current (here, $e > 0$). The change in the total current is positive because of the large increase in the condensate current (the sudden jump in I_{qp} at $v \simeq 1.45$ is inconsequential for the total current [4]). The transition to the AT regime is accompanied by an enhancement of the superfluid velocity (see figure 2(a)) and a depression
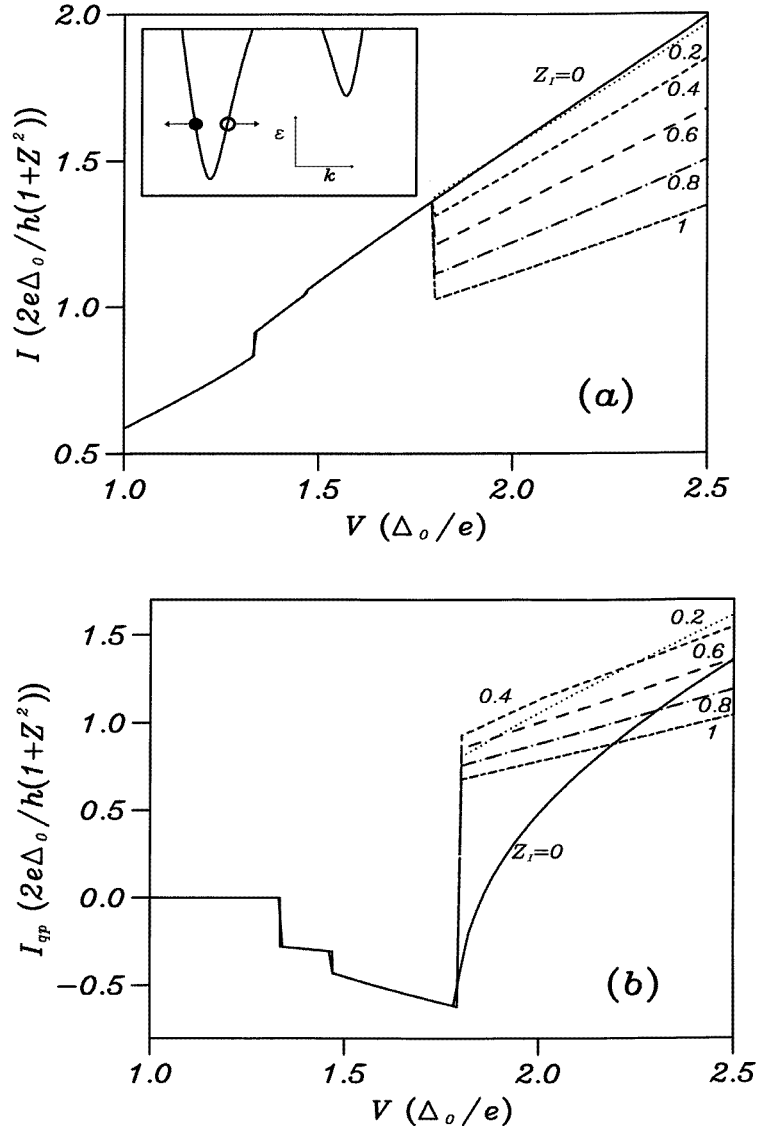
Figure 1. (a) Self-consistent I–V characteristics of an NSISN structure for $Z = 0.5$ and six different values of Z_I. The solid line corresponds to the case of a clean ($Z_I = 0$) NSN structure. Inset: schematic representation of the quasiparticle relation $\varepsilon(k)$ distorted by a nonzero v_s. Part (b) shows the quasiparticle component of the current.

of the gap (figure 2(*b*)). The regime of gapless superconductivity (GS) sets in for $v \simeq 1.8$, as can be guessed from the smooth increase in I_{qp}.

Let us now study the effect of adding an impurity ($Z_I \neq 0$) in the superconducting region. Although, for numerical simplicity, we present a one-dimensional calculation, the type of system we have in mind is a realistic three-dimensional superconductor with impurities or barriers that may cause scattering of quasiparticles. Thus, we neglect here any possible role of the impurity as a phase-slip centre. We rather view the impurity as a
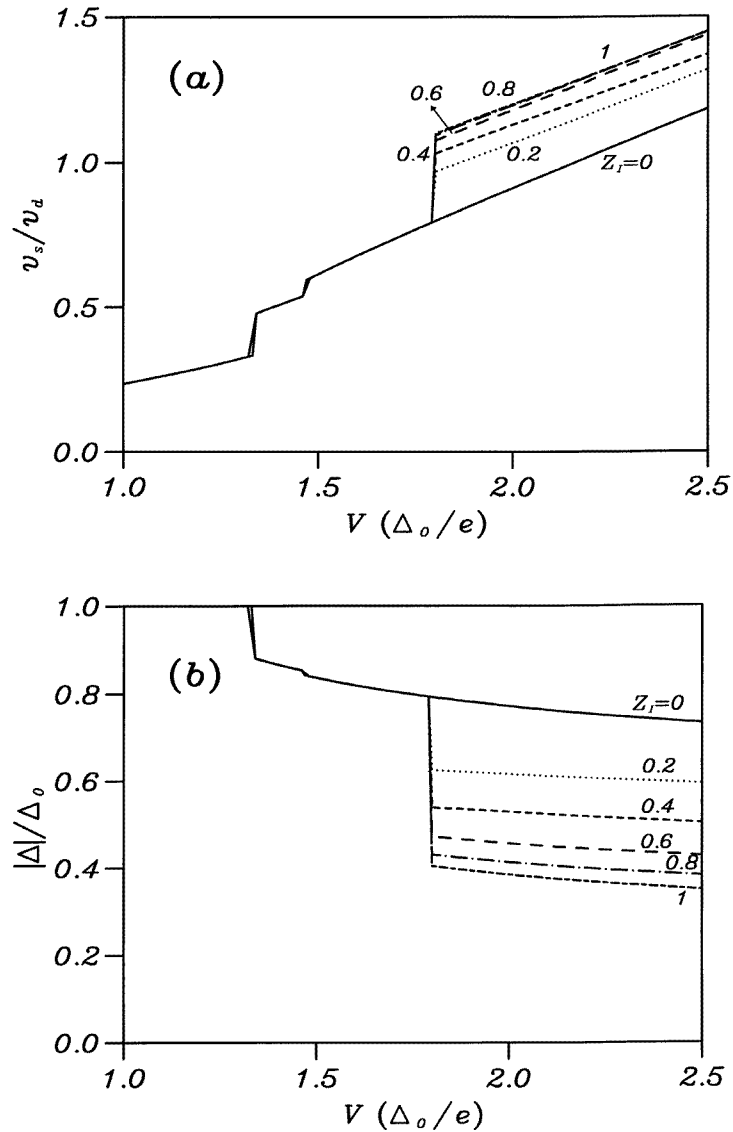
Figure 2. Same as figure 1, for the superfluid velocity (a) and the amplitude of the gap (b) in the superconducting region. v_s is given in units of the depairing velocity $v_d \equiv \Delta_0/\hbar k_F$.

mere source of incoherent quasiparticle scattering that affects the condensate only indirectly through the self-consistency condition (1).

At low voltages, charge transport is dominated by Andreev reflection. Current in the superconductor is entirely carried by the condensate and thus is not affected by impurity scattering. As in case of the clean structure, the AT channel opens at $v \simeq 1.3$, causing a marked increase in the total current. Due to the presence of quasiparticles one might naively expect the *I–V* characteristics to be sensitive to the presence of an impurity in the AT regime. However, the peculiar form of the quasiparticle dispersion relation makes

this presence ineffective. As already pointed out, in this regime only the low-lying states of the left branch are populated. Scattering by the impurity is unable to degrade the current carried by these quasiparticles, because only NT and AR are permitted and both mechanisms leave the sign of the electric current unaltered. Due to flux conservation and to the additional symmetry stemming from the two NS interfaces being identical, one finds that all the magnitudes depending on the quasiparticle population in the superconductor become impurity independent. This fact can be clearly appreciated in all the figures if one inspects the AT range $1.3 \gtrsim v \gtrsim 1.8$.

The situation changes dramatically when Δ_- becomes negative and the state of gapless superconductivity is established [4]. As the characteristic unconventional branch emerges at $k \simeq k_F$, the four quasiparticle channels become available at low enough energies. At this point, normal reflection by the impurity becomes possible and, with it, a strong degradation of the current. This fact is clearly observed in the $v \lesssim 1.8$ sector of figure 1(a). The dependence of the transport properties in the GS state on the impurity scattering strength Z_I constrasts markedly with the insensitivity found in the low-voltage regime: the stronger Z_I, the larger the decrease in the current. The decrease in the current is caused by both the appearance of NT into the unconventional gapless branch as the dominant current-carrying channel at the NS interfaces and the possibility of impurity-induced NR within the S region.

We also observe in figure 1(b) a sharp rise in I_{qp} that contrasts with the smooth behaviour of the clean system. These abrupt features in the current–voltage characteristics are also related to the jumps appearing in the self-consistently calculated values of the superfluid velocity and order parameter (see figure 2). The detailed nature of the sharp transition to the GS state seems to be very sensitive to the impurity strength. The reason why the size of the gap discontinuity increases with Z_I (see figure 2(b)) is similar to that which explains the behaviour of a clean NSN structure with high Z [4]. The population of quasiparticles placed in the emerging unconventional branch tends to reinforce the gap. In the presence of strongly reflecting barriers, however, the typical residence time of quasiparticles in the S region tends to increase and the resulting directional randomization tends to reduce $|\Delta|$.

One may wonder whether the physical effects discussed here would survive in a realistic structure with many transverse channels. Fortunately, the answer is yes, as can be inferred from the following argument (a numerical confirmation will be presented in [11]): the main complication caused by the presence of many transverse channels is that, for a given voltage, different channels may be in different transport regimes. This occurs because the shifted energy thresholds, $\Delta_\pm^{(n)} = \Delta_0 \pm \hbar q v_F^{(n)}$, depend on the index channel n ($v_F^{(n)}$ is the velocity available for the longitudinal propagation of Fermi electrons in channel n). Just above the low-voltage region dominated by AR, there must be a range of voltages in which scattering of quasiparticles into some channels within S (those with the highest values of $v_F^{(n)}$) is possible, but only by Andreev transmission. Even if impurities induce mixing among those modes, the basic effect of the absence of current degradation should remain, since the presence of many transverse channels does not change the fact that, in that voltage range, only right-moving quasiholes or left-moving quasielectrons are available. Therefore, one expects to observe the same kind of impurity independence that is found for the single-channel case.

Another potential problem is posed by the possibility that spatial variations in the effective quasiparticle chemical potential may induce a time-dependent response of the condensate. However, a gradient in the condensate chemical potential causes an increasing spatial variation of the phase that must be compensated by an unwinding mechanism. This requires the presence of phase-slip centres or a temperature very close to T_c [15], and both factors are ruled out here. The situation is rather one in which a charge imbalance develops

between the condensate uniform chemical potential and the nonequilibrium population of quasiparticles [16, 17]. The charge imbalance relaxation time, τ_ε, must be much longer than the average residence time $\tau_r \lesssim L/v_F$. On the other hand, $L \lesssim \xi_0$ is required for $|\Delta(x)|$ to reach its asymptotic value [5]. Thus, we require $\tau_\varepsilon \gg \hbar/\Delta_0$, which is physically realizable at sufficiently low temperature [18].

Finally, an important consequence of nonequilibrium is the absent of efficient heating. As in other experimental [19, 20] and theoretical [4, 5, 21] works, we find that voltage differences can coexist with superconductivity, even when eV is several times Δ_0. This is due to the lack of complete thermalization of quasiparticles in the S region.

From the analysis presented here we may envisage the following experimental scenario. A finite S segment is inserted in an N lead of the same or greater width. Contacts must be good to ensure that, at low temperatures, q becomes comparable to $\Delta_0/\hbar v_F$ when V is comparable to Δ_0/e. As in other mesoscopic transport contexts, voltage sources yielding thermal populations of incoming quasiparticles with a well defined chemical potential can be obtained by inserting the NSN structure in a wide circuit with low impedance.

At low voltages, AR dominates, the current is fully carried by the condensate and transport is insensitive to the impurity strength. As the voltage increases, a peak in the differential conductance signals the onset of quasiparticle Andreev tranmission. The AT regime is also insensitive to the presence of the impurity. At higher voltages, transverse modes with a high longitudinal Fermi velocity become gapless. The onset of GS should be clearly identified through a reduction in the current of a magnitude that increases with Z_I. This behaviour contrasts with that expected for structures in which the superfluid velocity remains negligible. That could be the case, for example, in an NSN structure in which the S segment is wider than the N leads. For such devices one expects that AT and conventional NT enter into action simultaneously at a voltage $V = 2\Delta_0(T)/e$, regardless of the value of Z. With all quasiparticle channels available, one expects the behaviour at voltages just above the first peak in dI/dV to be sensitive to the impurity strength Z_I.

In conclusion, in this paper we have studied the transport of current in an incoherent NSN structure with an added scattering source in the superconducting segment. Nontrivial effects appear when the current is large enough to make a self-consistent calculation necessary. The presence of a nonzero superfluid velocity creates a distortion in the quasiparticle dispersion relation that is responsible for the existence of several transport regimes. These regimes respond to the presence of impurities in different ways. In the regime where only Andreev transmitted quasiparticles are permitted in the superconductor, all the relevant magnitudes in the problem are essentially impurity independent. This happens because the impurity (or the set of impurities) is unable to normally reflect a quasiparticle and thus cannot degrade the electric current. At high voltages, when gapless superconductivity is reached, normal transmission at the interface and normal reflection by the impurity become possible, leading to a notable decrease in the total current that grows with the effective scattering strength.

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